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MODELLING COCNCRETE MATERIAL BEHAVIOUR UNDER HIGH LOADING RATES

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Abstract. *The present work sets out to investigate numerically, by means of dynamic Non-linear Finite-Element Analysis (NLFEA), the effect of the strain-rate exhibited under high loading rates (associated usually with blast and impact problems) on the material properties of structural concrete and the ensuing cracking process it undergoes. The numerical predictions obtained were initially validated against relevant published data obtained from tests on plain concrete prismatic and cylindrical specimens under increasing loading rates under direct or indirect tension, uniaxial compression and flexure. The numerical studies reveal that the responses under impact loading differ significantly from those under static loading once certain thresholds of loading rates are exceeded. The study builds on previous work [1-3], which has shown that the commonly observed stress-strain relationship of plain concrete specimens under high rates of loading actually describes dynamic structural response rather than material behaviour. A couple of finite element packages were used, in the present study, which adopt different approaches for modelling concrete material behaviour. The comparative analysis of the predictions obtained reveals that when realistically accounting for the brittle nature and the triaxiality which characterise concrete material behavior, the experi-*

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mentally and numerically observed variation in specimen behaviour (under increasing loading rates) is primarily attributed to parameters associated with structural response and not, as widely considered, to strain-rate sensitivity of the material properties of structural concrete. Furthermore, it is shown that strain-rate sensitivity in concrete is based on an interpretation of the experimental evidence through the use of material models the analytical formulation of which depends heavily on parameters associated with post-failure mechanisms (i.e. strain softening, tension stiffening, shear-retention ability, etc) which attribute ductile characteristics to concrete material behaviour that is not compatible with its brittle nature. The response is also affected by the nature of the problem at hand (a wave propagation problem within a highly nonlinear material). Therefore, it is concluded that at the material level the numerical study shows that the effect of high loading rates on the behaviour of concrete is mainly linked to the development of inertia forces rather than strain-rate sensitivity of its material properties. Overall the aim of the present study is to provide insight into the effect of loading-rate on the mechanics underlying RC structural dynamic response under impact loading.

1 INTRODUCTION

It has been well established, both experimentally and numerically, that the behaviour of plain concrete specimens (with a low moisture content) differs considerably from that under static loading, once certain thresholds of applied loading-rates are surpassed [4-7]. Nevertheless, there has been considerable debate concerning the reasons that trigger this shift. There is a consensus that inertia has a significant effect on the behaviour exhibited by plain concrete specimens under high-rate loading. However, considerable disagreement exists regarding the influence of strain-rate sensitivity on the material properties of structural concrete. Many researchers consider that the *material* properties of concrete are dependent on the rate of deformation (i.e. strain-rate dependency), a view which until recently was widely accepted and incorporated into the framework of existing military codes for the design and analysis of RC structures under blast and impact. Lately however, an increasing number of researchers [8-10] have expressed the opposite view, stating that the *material* properties of structural concrete (characterised by a low moisture content) are essentially strain-rate independent and that the observed changes in specimen behaviour with increasing loading rates is the result of (a) parameters affecting *structural* response, (b) the brittle nature and triaxiality characterising concrete material behaviour and (c) the nature of the problem at hand (a wave propagation problem within a highly nonlinear material).

The present study builds on previous research work [1-3], which has shown that the commonly observed stress-strain relationship of plain concrete specimens under high rates of loading actually describes dynamic *structural* response rather than *material* behaviour. The present study aims at investigating numerically, via dynamic nonlinear finite element analysis (NLFEA) of plain concrete prismatic specimens under increasing loading rates under uniaxial compression and tension. Emphasis is mainly focused on investigating the effect of loading rate on specific aspects of specimen behaviour, such as the maximum sustained load as well the variation of the stress-displacement within the specimen during the loading process. This is carried out in order to identify the true reasons that trigger the observed shift in specimen behaviour with increasing loading rates, which is a necessary prerequisite for developing an effective method (both in terms of safety and economy) for assessing RC structural response under high loading rates. Thus, the work will have implications for impact-resistant design.

For the numerical investigation, two finite element analysis (FEA) software packages were considered, both capable of carrying out three-dimensional (3D) nonlinear static and dynamic analyses, namely ABAQUS [11] and and RC-FINEL [12,13]. The latter study [1] is essentially assessed in the present work using ABAQUS. To realistically describe the nonlinear behaviour of concrete under high loading rates it is imperative that the material models, incorporated into any FEA package, are capable of accurately describing concrete material behaviour under both static and dynamic loading. ABAQUS incorporate concrete constitutive models the analytical formulation of which is based on continuum mechanics theories and relies heavily on the use of *post*-failure concrete characteristics (i.e. strain softening, tension stiffening and shear-retention). On the other hand, RC-FINEL incorporates a fully brittle empirical [13] material law describing concrete behaviour under triaxial loading while at the same time adopting a unique iterative solution strategy capable of compensating for the numerical instabilities stemming from the brittle concrete material law adopted [13]. The present and previous studies [1-3] have shown that the *material* properties of structural concrete are essentially strain-rate independent and that any effect of loading-rate on specimen behaviour is clearly attributed to parameters associated with the dynamic response of the specimen (acting as a *structure*). This sheds doubt on the applicability of such post-failure responses for the purposes of impact-resistant design.

2 MATERIAL BEHAVIOUR OF CONCRETE UNDER STATIC LOADING

Concrete is a heterogeneous material since its basic components (aggregate, cement paste and free water) have considerably different material properties [14]. Even in an unloaded state, it contains voids and discontinuities in the form of microcracks [13]. As a result, when an external load is applied on a concrete specimen, a complex internal stress field develops. The behaviour of concrete in uniaxial tension is mainly determined by testing plain concrete prisms in direct, or indirect (splitting) tension and flexure. Based on the available experimental data it is observed that concrete essentially behaves elastically until it reaches its tensile strength and then fails in a brittle manner suffering an abrupt loss of load carrying capacity. The behaviour of concrete under uniaxial compression is determined mainly by testing plain concrete cylinders or prisms (see Figure 1a) and is described through the typical stress-strain curves presented in Figure 1b. These curves describe the deformation behaviour of the concrete specimen parallel and normal to the direction of loading. Such curves comprise an ascending and a subsequent descending branch. Based on available experimental data [12,13,15-18], the descending branch does not describe *material* behaviour but the interaction between specimen and testing device. This interaction is the result of frictional forces developing at the interface between the specimen and the steel platens normally used to apply the external load. As a result the specimen is subjected to lateral confinement close to its supports (see Figure 1a). Another important aspect of concrete behaviour in compression is that it exhibits an abrupt increase of the rate of the lateral expansion when the load approaches the peak level. The latter level corresponds to the minimum volume level marking the beginning of a drastic volume dilation leading rapidly to failure. The variation of the volume of concrete under increasing compression is also shown in Figure 1b. Similar trends of behaviour are also observed under triaxial compression states of stress.

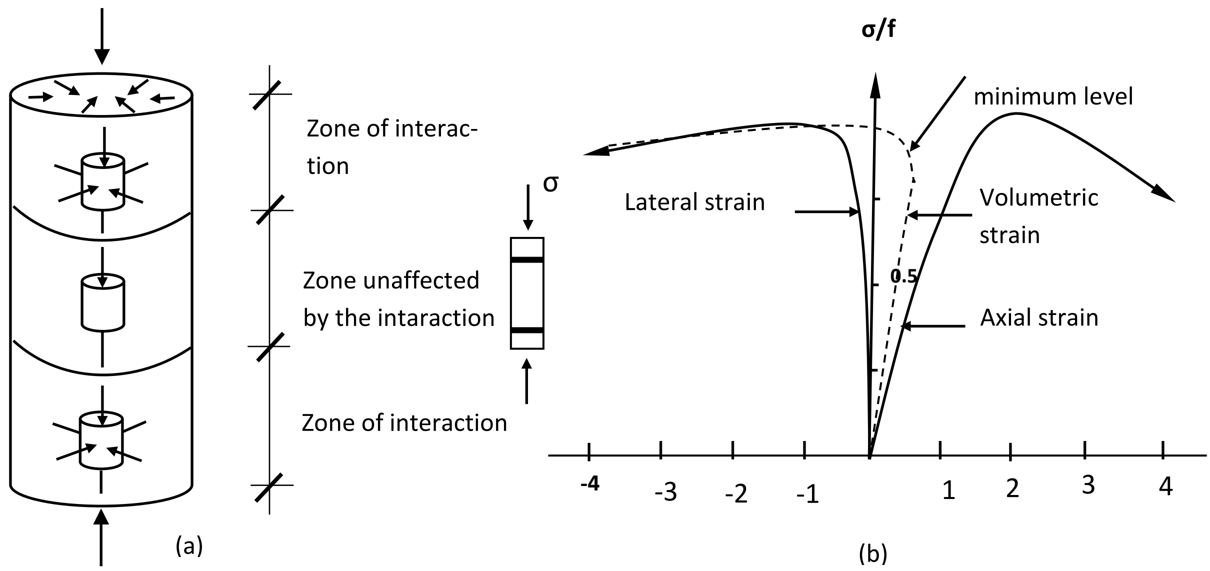
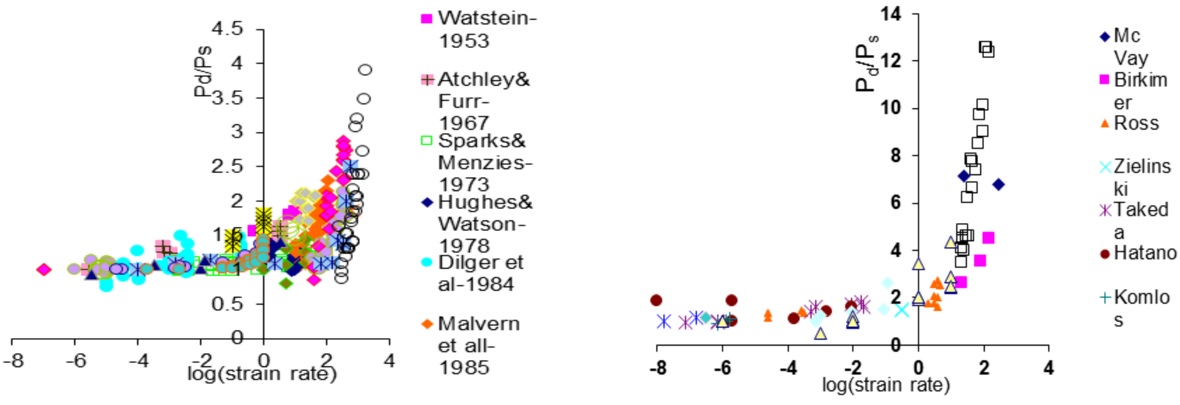


Figure 1: Stress state within the concrete cylinder specimen under uniaxial compression [13]

3 EFFECT OF LOADING RATE ON CONCRETE BEHAVIOUR

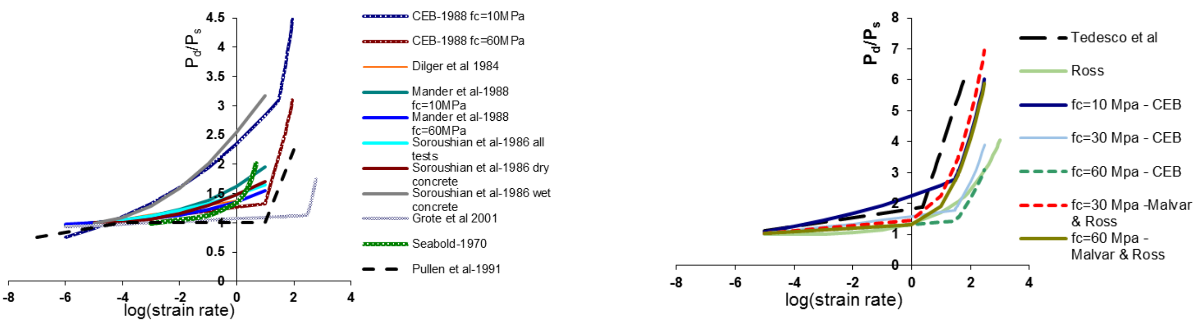
To date, a large number of experiments have been carried out to examine the behaviour of concrete (prismatic or cylindrical) specimens under high rates of uniaxial compression, direct or indirect tension [1,8,10]. The observed shift in specimen behaviour with increasing loading rates primarily takes the form of an increase in the maximum sustained load. This change becomes more pronounced with increasing loading rates and intensity. A summary of available experimental data is presented in Figure 2, expressing the relationship between the maximum sustained load maxP_d (normalised with respect to its load carrying capacity under static loading maxP_s) and the rate of axial defamation exhibited by the specimen expressed as strain-rate in compression (Figure 2a) or tension (Figure 2b).



(a) (b)

Figure 2: Experimental data for strength increase under different strain rates for concrete specimens under (a) compression and (b) tension [1]

Based on the available test data and the use of regression-analysis a number of curves have been proposed to date describing the increase of specimen strength under uniaxial compression (Figure 3a) and tension (Figure 3b) with increasing loading rates. However, it is evident that the behaviour of the concrete specimens under high-rate loading differs considerably compared to that established under static loading the experimental data cannot provide the reasons that trigger this shift. Furthermore, when inspecting the available experimental data it is clear that it is characterised by considerable scatter. This scatter is linked to a wide range of parameters which vary from experiment to experiment and include the experimental techniques implemented, the geometry and moisture content of the specimens, the different types of concrete used, etc [1,8]. As a result the available test data can only provide a *qualitative* description of the effect of loading-rate on the specimen behaviour.



(a) (b)

Figure 3: Regression-analysis curves for strength increase under different strain rates for concrete specimens under (a) compression and (b) tension [1]

In order to gain insight into the behaviour of concrete under high loading rates and to quantify the effects of various parameters on the dynamic response of plain concrete specimens, nonlinear finite-element analysis (NLFEA) was employed [19]. Through a comparison of the numerical predictions with the available experimental data, in a previous study, it was shown [1] that concrete specimens under impact loading must be viewed as *structures* and not as *material* units form which average material properties can be determined because their behaviour is directly linked to the inertia effect of their mass as well as to their geometry and the imposed boundary conditions. The present numerical study extends the previous one by using ABAQUS software.

4 CONSTITUTIVE MODELLING OF CONCRETE MATERIAL BEHAVIOUR

The formulation of analytical constitutive models describing the material behaviour of concrete is based mainly on the analysis of experimental data gathered from tests since the early 70's in which concrete specimens (cylinders or cubes) were subjected to uniaxial, biaxial or triaxial loading conditions. A review of the test data obtained is presented by [12, 13, 18]. Based on the preceding tests, stress-strain relationships are obtained describing the material behaviour under multiaxial stress-states as well as failure criteria. However, it should be noted that this data is characterised by considerable scatter due to heterogeneous nature of concrete as well as the variation of various parameters from test to test such as concrete mix, curing conditions, static uniaxial compressive strength of concrete f_c, the experimental techniques used, as well as the shape and size of the specimens [13].

The models of concrete behaviour may be broadly classified into two categories: those directly derived directly from regression analyses of experimental data (so-called empirical models) and those relying on continuum mechanics theories (such as, nonlinear elasticity, plasticity, visco-plasticity, damage mechanics) for their development, although the latter also remain dependent on the use of experimental data for their calibration. The resulting formulations usually incorporate a number of parameters, the evaluation of which is essential for achieving a close correlation between the model-predicted behaviour and its experimentally-established counterpart. These parameters, which are usually linked to post-peak strength concrete characteristics in both compression and tension (such as, for example, strain softening, tension stiffening, shear-retention ability) coupled with strain-rate sensitivity when high-rate loading conditions are considered, are often established through calibration based on the use of experimental information at the *structural*, rather than at the *material* level.

The application of FEA software packages in practical structural analysis has shown that the majority of constitutive relationships are case-study dependent, since the solutions obtained are realistic only for particular types of problems. The extension of the applicability of these packages to a different set of problems requires modifications, sometimes significant, of the constitutive relationships. The vast majority of constitutive models describing the behaviour of concrete under dynamic loading assume that there is a link between the constitutive properties of concrete and the strain rate at which the material is loaded and, consequently, the external loading rate. Although this seems to be the case for low loading rates, where creep plays a significant role in material behaviour, it has been proposed that for the case of high loading rates there is no need to change the static value of Young's modulus [1,7-10].

4.1 RC-FINEL and ADINA

RC-FINEL has been found to provide realistic predictions of a wide range of different concrete structural forms under static (monotonic and cyclic) [13] and dynamic (earthquake and impact) [12] loading conditions. It incorporates a brittle material model, which describes the behaviour of concrete under triaxial loading conditions [13], as well as a unique nonlinear strategy the formulation of which allows for the brittle nature of the material model employed, while at the same time it provides a realistic description of the cracking process and minimizes the likelihood of numerical instabilities associated with this process [10-12]. The material model of concrete behaviour adopted is characterised by both simplicity (fully brittle, with neither strain-rate nor load-path dependency, fully defined by a single material parameter - the uniaxial cylinder compressive strength f_c) and attention to the actual physical behaviour of concrete in a structure (unavoidable triaxiality which is described on the basis of experimental data of concrete cylinders under definable boundary conditions).

Crack formation is modelled by using the smeared-crack approach. A crack forms when the stress developing in a given part of the structure corresponds to a point in the principal stress space that lies outside the surface defining the failure criterion for concrete, thus resulting in localized material failure. This failure takes the form of a crack and is followed by immediate loss of load-carrying capacity in the direction normal to the plane of the crack. Concurrently, the shear stiffness is also considered to reduce drastically to a very small percentage of its previous (i.e. uncracked) value. However, it is not set to zero in order to minimize the risk of numerical instabilities during the execution of the solution procedure, as explained elsewhere [12,13]. A procedure for crack closure is also provided and full description of the triaxial constitutive model and numerical formulations used in RC-FINEL software is provided elsewhere [12,13]. It should be noted that ADINA software package [20] now includes the same triaxial model by Kotsovos and Pavlovic [13] that is used in RC-FINEL, although there some differences in the non-linear solution strategy.

4.2 ABAQUS

The concrete damaged plasticity model is offered in ABAQUS for the analysis of concrete at low confining pressures (less than four or five times the ultimate compressive stress in uniaxial compression loading). It is based on the models proposed by Lubliner et al. [21] and Lee and Fenves [22], with the assumption of scalar (isotropic) damage and is designed for applications in which the concrete is subjected to arbitrary loading conditions. The model takes into consideration the degradation of the elastic stiffness induced by plastic straining both in tension and compression. There are two failure mechanisms for the mechanical response of the concrete include tensile cracking and compressive crushing. The evolution of failure is controlled by two hardening variables, which are referred to as equivalent plastic strains in tension and compression, respectively. The model assumes that the uniaxial tensile and compressive response of concrete is characterized by damaged plasticity, adopting a post-peak branch on the corresponding stress-strain curves.

5 RESULTS AND DISCUSSION

5.1 Dynamic problem investigated

The present numerical investigation uses a concrete prism, similar to the specimens usually tested to determine the material properties of concrete under static loading. The prisms were subjected to uniaxial compression and tension, for which the prisms were assumed to be fixed at their bottom face and are subjected to an axial load applied at their upper face, through a rigid element (mimicking steel platen in experiment) with the same cross-section (see Figure 4) in order for the external load to be distributed uniformly on the upper face of the concrete prism. It is assumed that the concrete prismatic specimen and the rigid element on top are fully bonded at their interface. The prism height is 253mm and its cross-section forms a square with a width of 100mm whereas the rigid element has a height of 200mm. Because of the double symmetry of the problem at hand, one quarter of the actual specimen was modeled with suitable boundary conditions. The uniaxial compressive strength of concrete f_c is 30MPa. In all cases the external load is applied incrementally as a force with its value increasing linearly at a constant rate until the load-carrying capacity of the RC specimen is reached and failure occurs. Initially the static problem is investigated to effectively calibrate the model and this is followed by the investigation of the dynamic problems at higher loading rates.

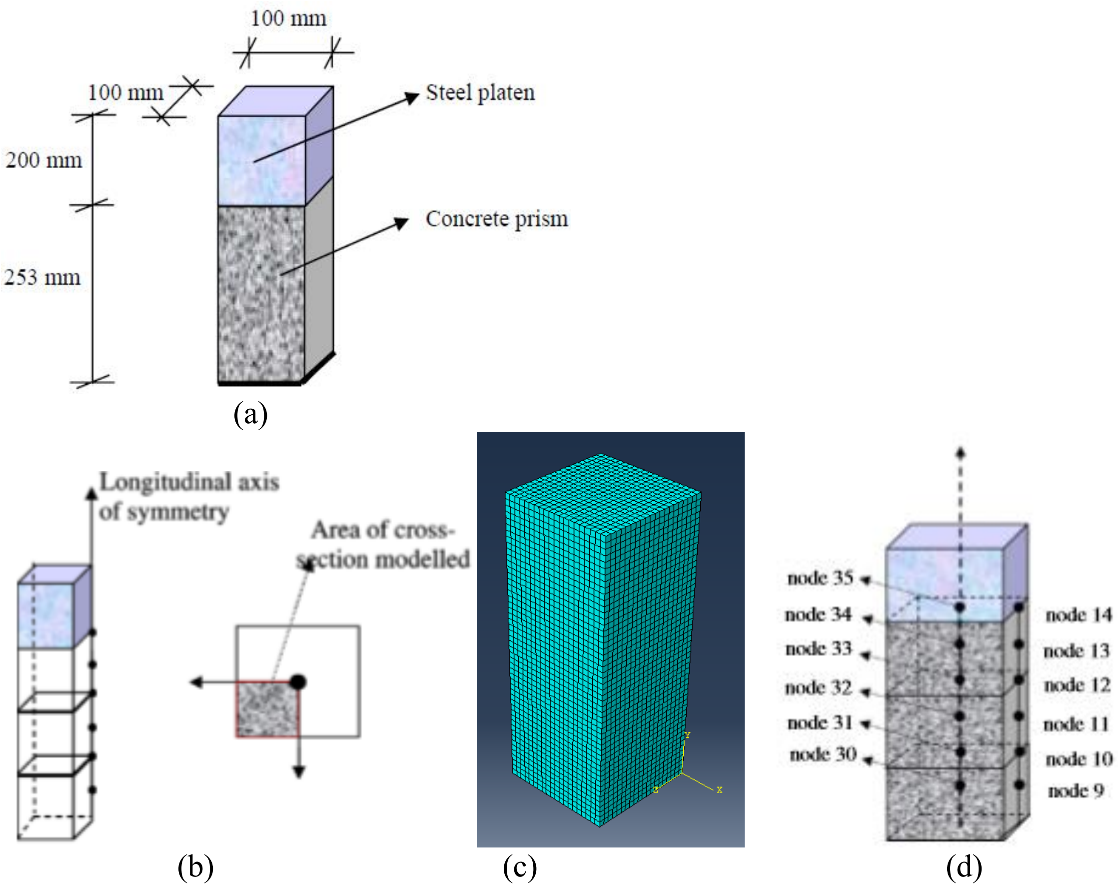


Figure 4: (a) Details of the prism specimen modelled, (b) symmetrical arrangement and coarse mesh adopted by Cotsovos and Pavlovic [1], (c) finer mesh used in current study and (d) nodes where values of axial (30-35) and lateral (9-14) displacements were examined

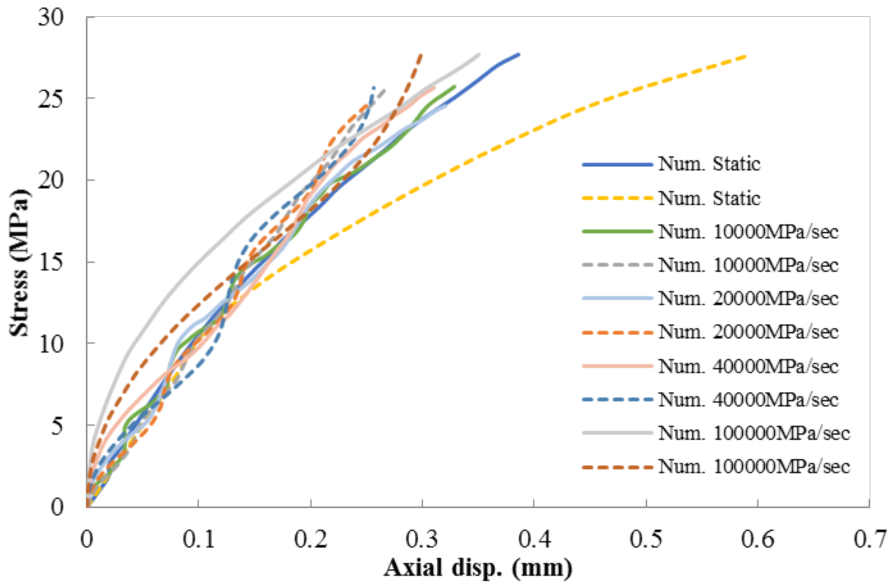
5.2 Modelling of the dynamic problem

In the previous numerical study by Cotsovos and Pavlovic [1], a coarse mesh of three 27-noded brick elements (see Figure 4b) was used. This follows the philosophy upon which the FE model used in that study was based, which does not employ small finite elements [13]. This is because the latter material model is based on experiments on concrete cylindrical specimens (subjected to various triaxial loading conditions) and thus these cylinders constitute a "material unit" for which average material properties are obtained. Hence the volume of these cylinders was used to provide a reasonable guideline to the order-of-magnitude of the size of the FE which should be used for concrete modelling. In the present study, on the other hand, the same concrete specimen was modelled using a finer mesh with an element size of 5 mm as shown in Figure 4c. A different constitutive model for concrete was also used, i.e. plasticity-based as opposed to the brittle one used previously. The present study aims to investigate the capability of the FE model developed using ABAQUS software [11] to simulate the behaviour of concrete prism under high rates of axial compressive and tensile loading. A reduced integration scheme is adopted for the element formulation to avoid numerical problems due to locking. The load was applied as uniform pressure directly on the prism top surface.

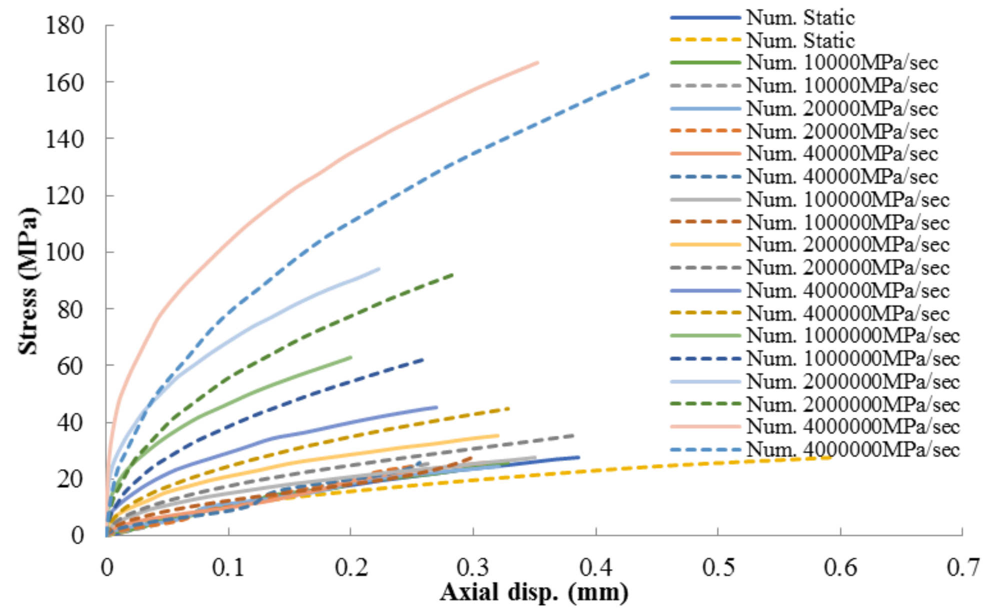
5.3 Discussion of results under high rate of compressive loading

A comparison was carried out between the previous study (adopting macro modelling) using FINEL software [1] and the present research work (utilising much smaller elements) using ABQUS software, under increasing rates of both compressive and tensile loading. The results for the compressive loading are depicted in Figure 5, which show that the axial stress-displacement curves for both studies are in good agreement. It can also be seen that the response of the concrete prism under static and low rates of loading (lower than 200,000 MPa/sec) are similar. But, the numerical stress-displacement curves begin to differ from their static counterparts as the applied loading rate increased (above 200,000 MPa/sec). It can be concluded that, as the rate of loading becomes higher, the load-carrying capacity and sustained maximum axial strain exhibited by the specimen increase.

The variation of the load-carrying capacity maxP_d (normalised with respect to its value under static loading maxP_s) at the maximum sustained load versus the stress and strain rates are shown in Figure 6(a) and (b), respectively.



(a)



(b)

Figure 5: Comparison between numerical stress-displacement curves from previous study [1] (continues line) with current numerical study (dashed line) for concrete prism under different stress rates up to (a) 100,000 and (b) 4,000,000 MPa/sec

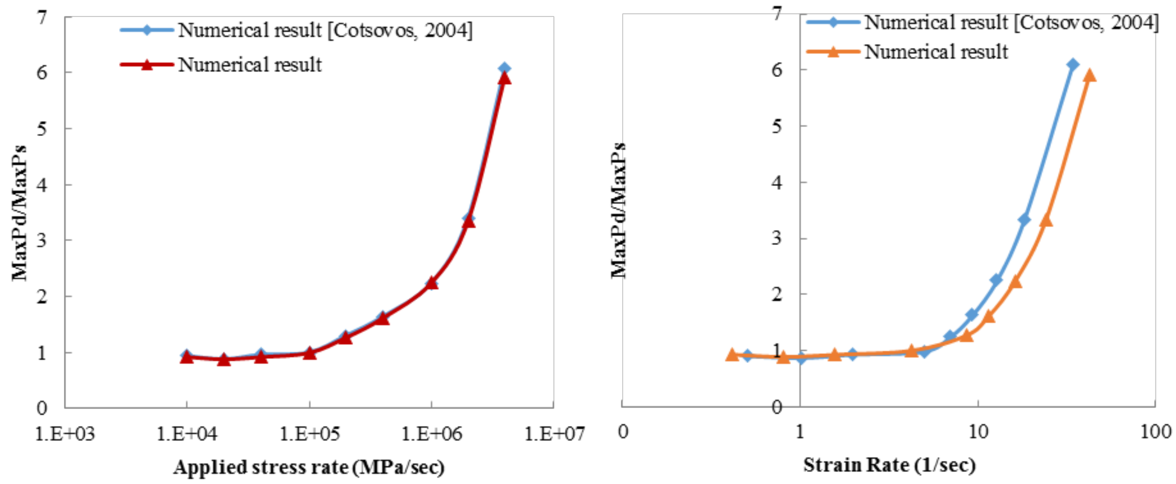
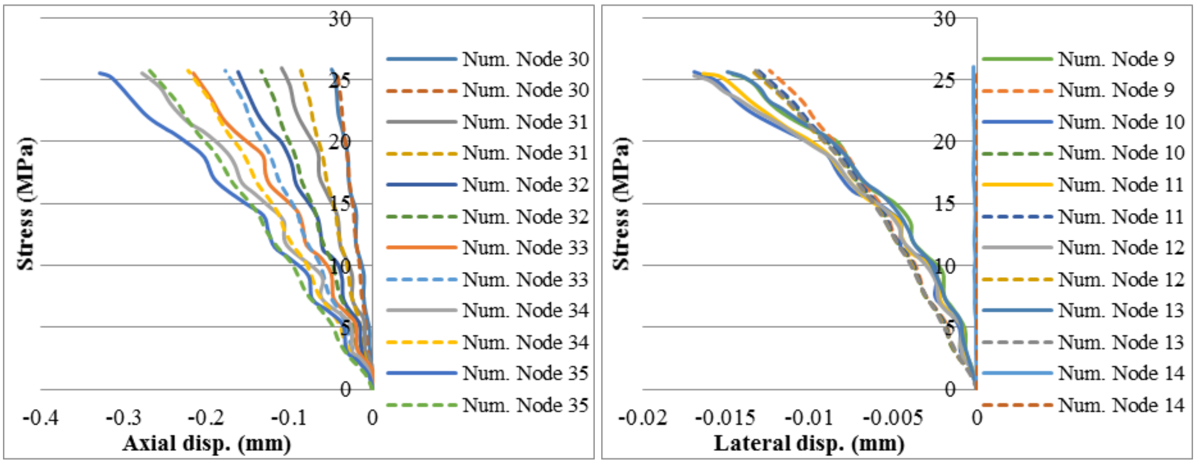
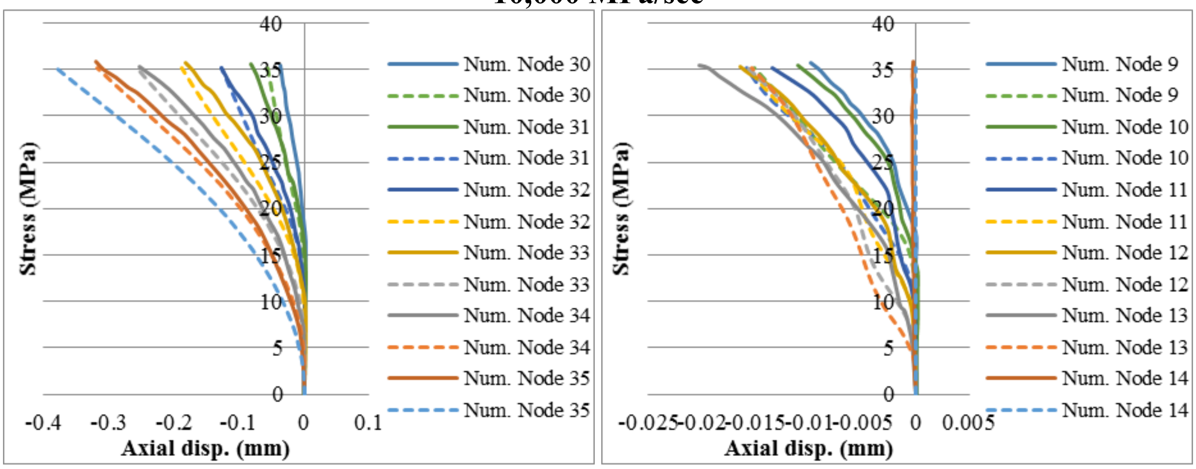


Figure 6: Comparison of the numerical DIF expressed as (a) maxP_d/maxP_s and (b) maxε_d/maxε_s for different applied stress rate based on previous study [1] and current numerical study

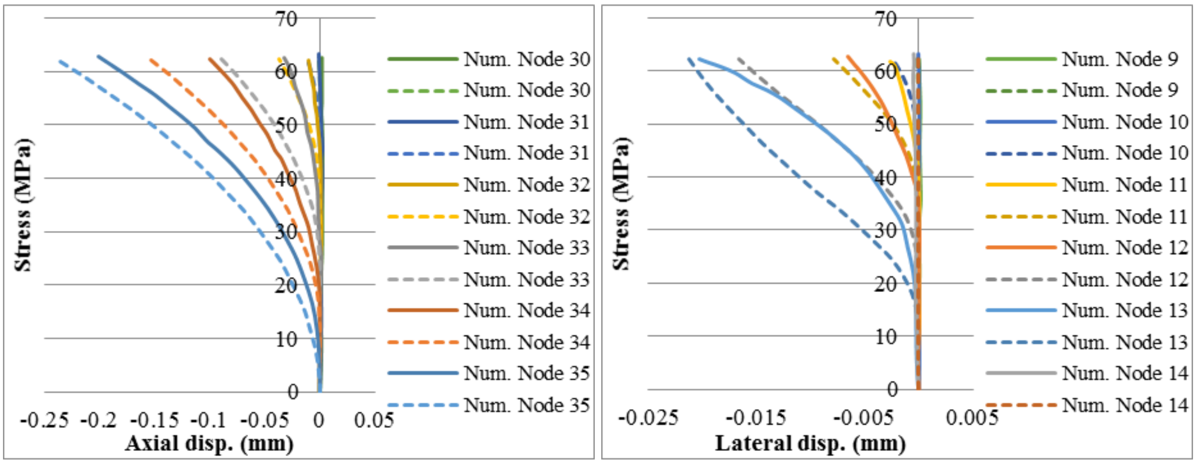
Axial and lateral displacement were measured at different nodes along the height of the specimen and the relevant nodes are highlighted in Figure 4d. The ensuing numerical results presented in Figure 7 show that both axial and lateral displacements of the nodes close to the bottom of the specimen gradually decrease as the rate of loading increases (over the value of 200,000 MPa/sec). This displacement becomes very small (even negligible) as compared with the displacements at the upper part of the specimen in the case of very high rates of loading (over 400,000 MPa/sec). In addition, the numerical results presented in Figure 7 show that the axial displacement in static case is linear, while the displacement at the nodes in the lower part of the specimen becomes negligible in the case of high rate of loading. This was true for both the previous and current numerical studies, which are in good agreement. Figure 8 depicts a schematic representation of the axial and lateral deformations under both low- and high-rate loading. Figure 8 indicate that, as the rate of loading increases, the portion of the height (h_{eff}) of the prism mostly affected by the applied load reduces. For relatively high rates of loading, this *effective* length is confined in the region of the prism in the proximity of the applied load (at the upper face) and extending downwards, whereas the remainder of the specimen (i.e. the bottom portion) remains practically unaffected by the applied load. Therefore, under high rates of loading, the prism behaviour is essentially characterized by h_{eff}. This explains the apparent increase in load-carrying capacity and confirming that it is not an intrinsic *material*, but is due to the specimen responding as a *structure* with different *effective* lengths.



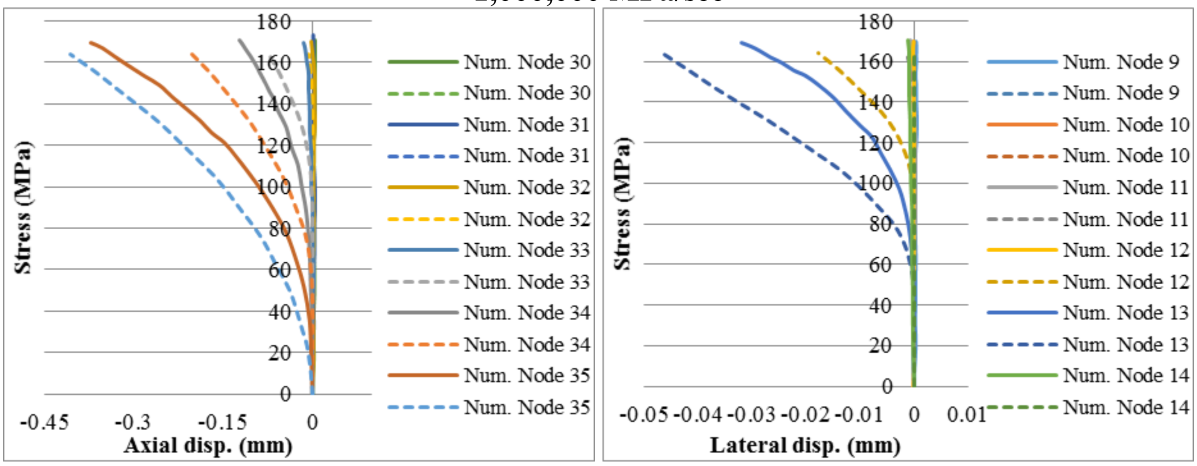
10,000 MPa/sec



200,000 MPa/sec

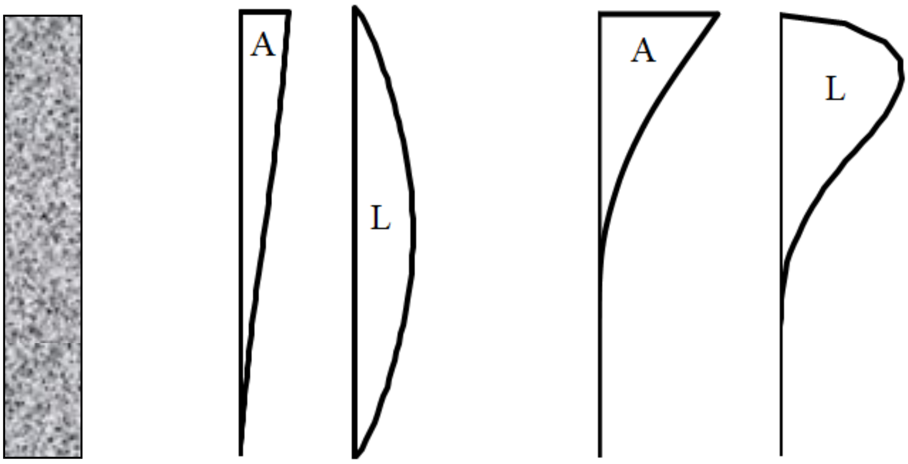


1,000,000 MPa/sec



4,000,000 MPa/sec

Figure 7: Comparison of numerical axial stress-axial displacement and -lateral displacement curves at different stress rates, with the results measured at different distances from the top face of the specimen based on previous study [1] (continues line) and current numerical study(dashed line)



Static **High rate of loading (2,000,000 MPa/sec)**

Figure 8: A schematic representation of the distribution of axial displacement (A) and lateral displacement (L) along the longitudinal axis of the concrete prism prior to failure

5.4 Discussion of results under high rate of tensile loading

Similarly to the case with compressive loading, the responses of the specimen under tensile loading (see Figure 9) shows that the axial stress-displacement curves of the concrete prism under static and low rates of loading (lower than 200,000 MPa/sec) are similar. At higher loading rates, on the other hand, curves begin to differ from their static counterparts as the applied loading rate increased. Thus, it can be concluded that, as the rate of loading becomes higher, the load-carrying capacity and sustained maximum axial strain exhibited by the specimen increase. This is confirmed in Figure 10, which shows the variation of the axial reaction with the stress rate. The results of the previous FINEL-based and the current ABAQUS-based study were also in good agreement.

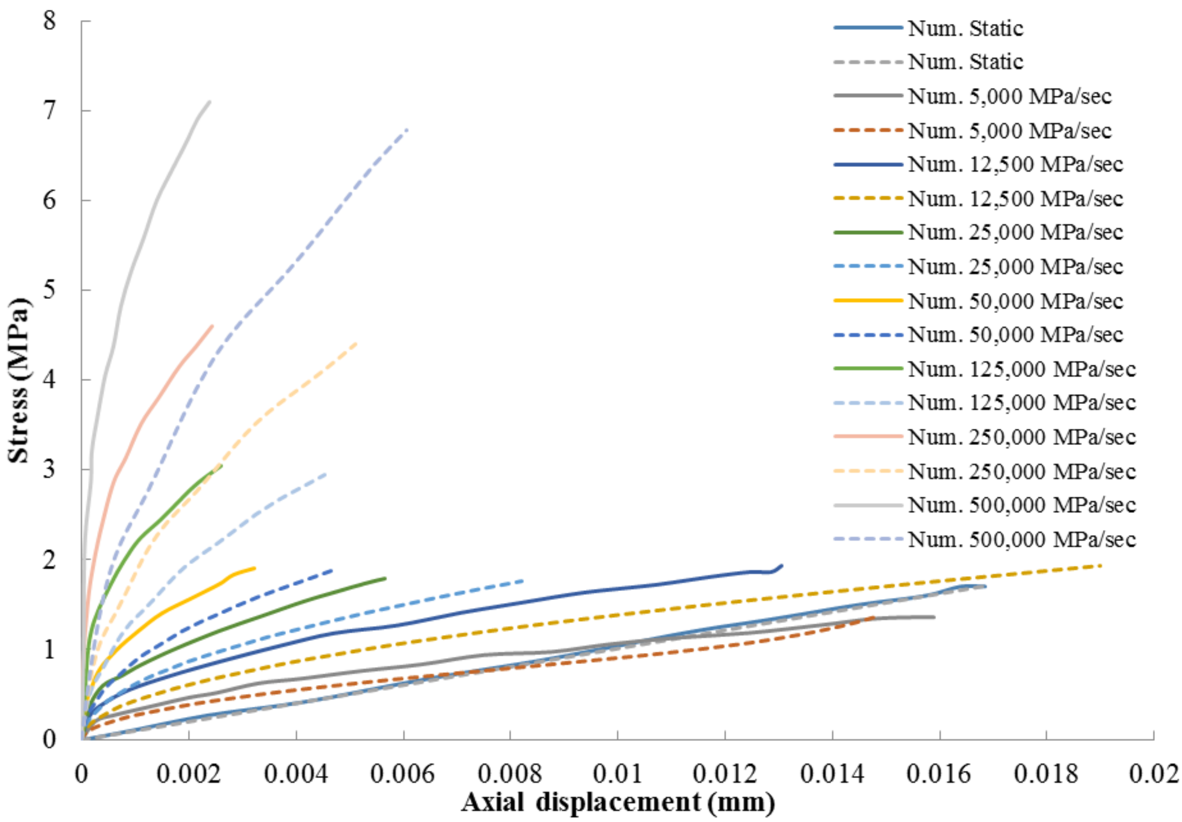


Figure 9: Comparison between axial tensile stress-axial displacement curves for concrete prism under different stress rates based on previous study [1] (continues line) with current numerical study (dashed line)

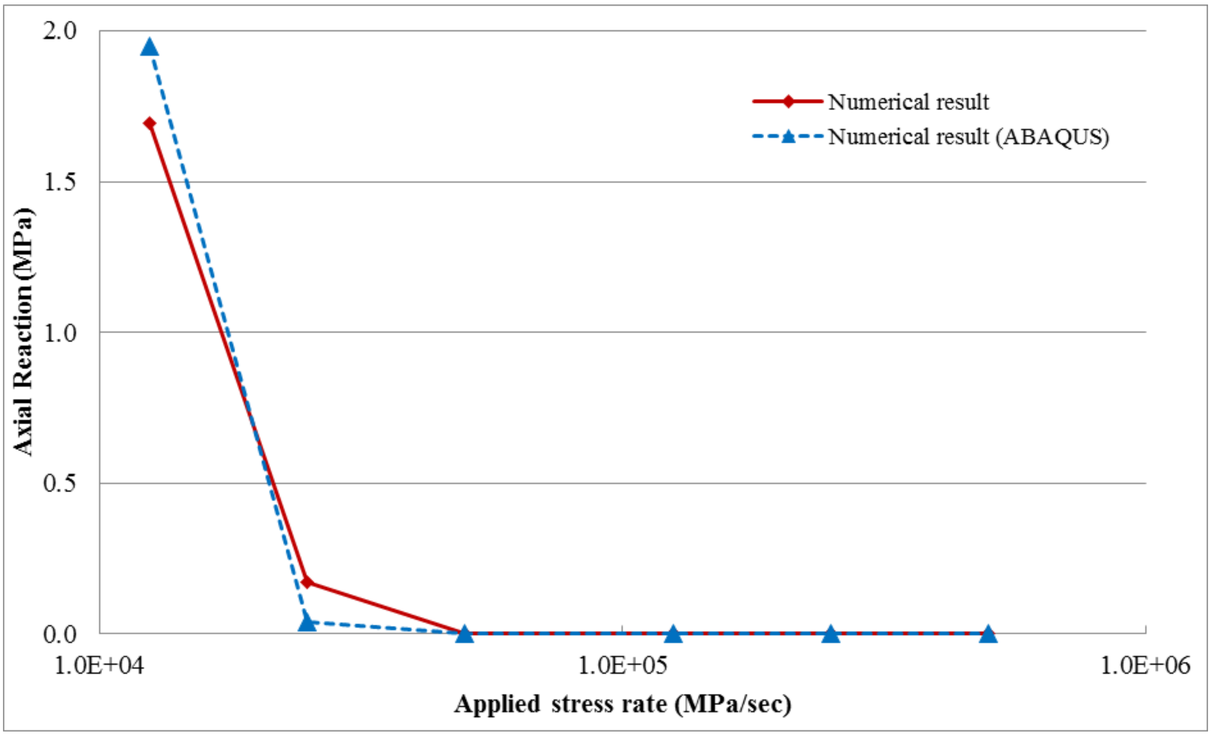


Figure 10: Comparison of the numerical results of the variation of the axial reaction with stress rate at the bottom of the specimen (dashed line) with previous study [1] results (continues line)

6 CONCLUSIONS

- Concrete specimens under impact loading need to be viewed as *structures* and not as *material* units form which average material properties can be determined (i.e. only correct under static loading).
- Specimen behaviour is directly linked to the *inertia* effect of their mass as well as to their geometry and the imposed boundary conditions.
- Impact problem should be considered as a wave propagation problem within a highly nonlinear material as the application of the external loads leads to the development of stress waves which propagate through the concrete medium, away from the area where the load is imposed.
- The propagation speed of these waves depends on the material properties of concrete and the level of damage (cracking) sustained by the concrete medium, while their intensity level depends on the rate of loading and the magnitude of the applied load.
  - In the case of low loading rates, the stress waves generated, travel through the whole length of the specimen.
  - Under high rates of loading the numerical results reveal that the specimen exhibits a *localised* response as failure occurs prior to the stress waves being able to travel the full length of the specimen.
  - As a result *only* the region of the concrete specimen close to the point of loading deforms whereas the rest remains practically unaffected.

- Under high rates of uniaxial compression, high rates of axial and lateral deformation are also exhibited which trigger the development of significant *inertia* forces in these directions. These forces have a *confining* effect on the concrete prism and tend to restrict the deformations both axially and laterally, thus essentially confining the concrete specimen (*inertial confinement*) and slowing down the cracking process it undergoes compared to that established under static testing.
- The confinement effect allows the prism to exhibit higher load-carrying capacities and maximum values of strain compared to those established under static loading.

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